EXAMINER: Dr M F Thomas
DEPARTMENT: Physics Tel No: 43354

THE UNIVERSITY
*of* LIVERPOOL

# JANUARY 2006 EXAMINATIONS

Degree of Bachelor of Science: Year 3
Degree of Master of Physics: Year 4

## STATISTICAL AND LOW TEMPERATURE PHYSICS

TIME ALLOWED: THREE HOURS

---

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Question 1 carries 50% of the total marks.

Question 2 and 3 each carry 25% of the total marks.

The marks allotted to each part of a question are indicated in square brackets.

In the event of a student answering both parts of an either/or question and not clearly crossing out one answer, only the answer to part (a) of the question will be marked.

---

1.

(a). A set of 4 distinguishable particles occupies energy states 0, ε, 2ε, 3ε, 4ε ......
The total energy of the set is 5ε.

(i) Write out the 6 possible distributions [2]

(ii) Evaluate the number of microstates for each distribution [2]

(iii) Evaluate the mean populations of the states. [2]

(b) N atoms bound into a solid system at temperature T can each exist in states of energy 0, ε, 2ε.

(i) Write an expression for the Partition function of the atoms. [2]

(ii) Using the bridge relation

$$U = NkT^2 \frac{\partial (\ln Z)}{\partial T}$$

or otherwise, show that the internal energy U can be written as

$$U = \frac{N\varepsilon.\exp(-\varepsilon/kT).[1+2\exp(-\varepsilon/kT)]}{[1+\exp(-\varepsilon/kT) + \exp(-2\varepsilon/kT)]}$$ [2]

(iii) Derive the limiting values of U as $T \to 0$ and as $T \to \infty$. [2]

(iv) Sketch a graph of U versus T. [2]

(v) Without further differentation sketch the graph of $C_V$ versus T. [2]

(c) The Maxwell-Boltzmann distribution of speeds of molecules n(v) in a box containing N molecules of mass m at temperature T is written

$$n(v) = 4\pi N.(m/2\pi kT)^{3/2} v^2 \exp(-mv^2/2kT)$$

(i) Draw the distribution n(v) versus v. [2]

(ii) Derive an expression for the most probable velocity $v_p$. [2]

(iii) Derive an expression for the mean square speed $v^2_m$ [2]

(iv) Estimate the energy of a mole ($6x10^{23}$ molecules) of a monatomic gas at room temperature (300K). [2]

$$\text{Integrals } I_n = \int_0 x^n \exp(-bx^2)$$

Can be evaluated from

$$I_n = [(n-1)/2b]\, I_{n-2}, \qquad I_0 = \tfrac{1}{2}(\pi/b)^{1/2}, \qquad I_1 = 1/2b$$

(d) The molecule HD is composed of an atom of hydrogen (H, atomic mass = 1) bonded to an atom of deuterium (D, atomic mass = 2).
Vibrational energy levels of such molecules are given by

$$E(n) = (n+\tfrac{1}{2})\, \hbar\, (B/\mu)^{1/2}$$

where B is the inter-atomic force constant, $\mu$ is the reduced mass of the molecule and n is an integer.
Rotational energy levels are given by the relation

$$E(I) = \frac{\hbar^2 I(I+1)}{2\mu r^2}$$

where the integer I is the quantised angular momentum and r is the separation of the atoms.
(i) Evaluate excitation energies for the first excited states of vibrational and rotational motion for the HD molecule in which $B = 6.91\mathrm{x}10^2$ kg.s$^{-1}$ and $r = 1.05\mathrm{x}10^{-10}$m. **[2]**
(ii) Estimate temperatures $\theta_V$ and $\theta_R$ at which vibrational and rotational motions become populated for the HD molecule. **[2]**
(iii) Give reasoned estimates for the heat capacity $C_V$ of a mole of HD at temperatures T = 20K and at T = 300K. **[2],[2]**

(e)
(i) Draw diagrams depicting the constituents of an atom of $He^3$ and an atom of $He^4$. **[2]**
(ii) In each case specify the electronic, nuclear and total angular momentum values of the atom **[2]**
(iii) Explain why one type of atom is a fermion and the other is a boson. **[2]**
(iv) $He^3$ atoms in liquid form at T = 2K in a container occupy a level scheme of quantised states. Sketch the occupation of these levels. **[2]**
(v) $He^4$ atoms in liquid form at T = 2K in a container occupy a level scheme of quantised states. Sketch the occupation of these levels. **[2]**

(f) The critical field $B_C(T)$ in a superconductor at temperature T is related to that at T = 0, $B_C(0)$, by the relation

$$B_C(T) = B_C(0).[1-(T/T_C)^2]$$

where $T_C$ is the critical temperature at B = 0.
(i) Sketch the relation $B_C(T)$ versus T. **[2]**
(ii) Label the regions of superconducting and normal behaviour on the sketch **[2]**
(iii) The element lead has $T_C = 7.2$K and $B_C(0) = 0.080$T. Calculate the critical temperature for lead in a field of B = 0.05T. **[2]**
(iv) Is lead at a temperature of T = 5.6K in an applied field of B = 0.035T in a superconducting or a normal state? **[2]**

2. Answer **either** 2(a) **or** 2(b)

2(a) A system is composed of N conduction electrons which move freely within a cube of metal of side L.

(i)Write quantised values for the wavevector components $k_x$, $k_y$, $k_z$ corresponding to allowed motions of the electrons. [2]

(ii) Illustrate the allowed states in $k_x$, $k_y$, $k_z$ space. [2]

(iii)Taking account of the spin degeneracy of electrons, show that the number of states g(k) with values of k in the range k to k +dk is

$$g(k)dk = \frac{2.V.4\pi k^2.dk}{(2\pi)^3}$$

where the volume $V = L^3$. [4]

(iv) Write the relation between k and energy ε. [1]

(v) Show that the number of states with energy in the range ε to ε + dε is

$$g(\varepsilon)d\varepsilon = \frac{2.V.(2m/\hbar^2)^{3/2}.\varepsilon^{1/2}.d\varepsilon}{(2\pi)^2}$$ [3]

(vi) The probability that a state with energy ε will be occupied is f (ε).
Sketch graphs of f (ε) versus ε for electrons
For temperature T = 0
For temperature $0 < T < T_F$ where $T_F$ is the Fermi temperature.
Indicate the Fermi energy μ(0) at T = 0. [3]

(vii) Show that μ(0) is given by

$$\mu(0) = (\hbar^2/2m).(3\pi^2 N/V)^{2/3}$$ [4]

At temperature T the internal energy U of the electrons can be written

$$U = \frac{2N\mu(0)}{3} + \frac{N\pi^2.(kT)^2}{4\mu(0)}$$

Metallic silver has a molar volume of $10.27 \times 10^{-6}$ $m^3$ and each atom contributes one electron to the conduction band.

(viii) Evaluate the Fermi energy μ(0) for silver. [3]

(ix) Evaluate the electron heat capacity $C_V$ for a molar quantity of silver at a temperature of 5K. [3]

2(b)

(i)Describe what is meant by black body radiation. **[4]**

The density of states g(k) in terms of wave-vector k for quantised electromagnetic waves in a cavity of volume V is written as

$$g(k)dk = 2.\frac{V.4\pi k^2\, dk}{(2\pi)^3}$$

(ii) Show that this density of states can be written in terms of the frequency ν of the waves as

$$g(\nu)d\nu = \frac{8\pi V\nu^2\, d\nu}{c^3}$$

where c is the velocity of light. **[3]**

(iii)The energy contained in the frequency interval ν to ν + dν of the radiation is given by

$$\varepsilon(\nu)d\nu = \frac{8\pi V\nu^2 d\nu}{c^3} . h\nu . \frac{1}{[\exp(h\nu/kT - 1]}$$

Explain the meaning of the factors hν and $\frac{1}{[\exp(h\nu/kT) - 1]}$ in this expression. **[2]**

(iv)Deduce the limiting values of ε(ν) as $\nu \to 0$ and as $\nu \to \infty$. **[2],[2]**

(v)Sketch the distribution of ε(ν) versus ν and relate the energy density (U/V) to the sketch. **[4]**

(vi) Using the integral given below – show that

$$(U/V) = \frac{8\pi^5 . (kT)^4}{15 . (hc)^3}$$

**[4]**

(vii) Evaluate the energy density at T = 1000K **[2]**

(viii) Calculate the temperature at which the energy density is 10 times greater than it is at T = 1000K. **[2]**

$$\frac{y^3\, dy}{[\exp(y) - 1]} = \frac{\pi^4}{15}$$

Answer **either** 3(a) **or** 3(b).

3(a)

(i)Describe the basic features of the microscopic theory of superconductivity. **[4]**

Use this theory to explain:

(ii) the nature of the current carriers in the normal and superconducting states **[2]**

(iii) the critical temperature $T_C$ and the critical field $H_C$ of a superconductor **[3]**

(iv) the isotope effect **[3]**

(v) the comparison between the resistivity of a non-superconducting metal (Cu) and the normal phase of a superconducting metal (Pb). **[3]**

(vi) the nature of high $T_C$ superconductors **[3]**

(vi) Describe the Meissner effect. **[3]**

(viii) Draw graphs to illustrate the behaviour of Type I and Type II superconductors **[2]**

(ix) Give an example of a Type I and a Type II superconductor. **[2]**

3(b)

Draw the pressure versus temperature phase diagram for $He^4$ and label the different phases. **[4]**

Describe a model for liquid He II and show how this model explains the measurements viscosity for this liquid phase. **[4]**

Draw the pressure versus temperature phase diagram for $He^3$ and label the different phases. **[4]**

Does liquid $He^3$ show superfluid properties? If so give a brief description of how these may be explained. **[2]**

Why may parts of the phase diagram be sensitive to the application of a magnetic field? **[2]**

Describe **one** method of attaining temperatures of about 1mK using liquid helium. The description should include the type of helium used, the basic theory of the method, a schematic sketch of the apparatus and the starting temperature of the process. **[9]**